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SOME PROBLEMS OF HEAT AND MASS EXCHANGE OF A GAS SUSPENSION WITH RECUPERATIVE ENERGY SUPPLY

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The problem of heat exchange between a stream of a gas suspension and a third heat-transfer agent with recuperative energy supply is formulated and solved numerically.

The field of application of apparatus containing streams of a gas suspension with conductive heat supply is extensive. It is sufficient to cite heat-exchange apparatus for drying (pneumatic pipes with outside heating and cooling), recuperators, and special cases of intensification of the heat-transfer process using an intermediate disperse packing. Some types of apparatus of this kind are presented in [1]. In the present report we discuss apparatus containing coaxially arranged cylinders: a gas suspension moves through the inner channel and a heat-transfer agent (gaseous or liquid) moves through the annular outer channel (Fig. 1a). Consequently, we are talking about heat exchange between three heat-transfer agents. Various cases of heat exchange between three heat-transfer agents with a constant heat-exchange surface are discussed in [2], andwith variable heat-exchange surfaces in [3]. In the latter case disperse particles are used as two of the heat-transfer agents. A special feature of the calculation of such systems is that the surface of each of the disperse heat-transfer agents is a variable quantity (in time or along a coordinate). Since the laws of variation of the surfaces are the same, their ratio is represented in the form of a linear dependence, which simplifies the solution of the problem. When a drying process or a drying process combined with cooling is being performed, most often only one heat-transfer agent is disperse (the material being dried or cooled), and its surface is a variable quantity; the second heat-exchange surface, the surface of the cylinder, while it varies with the coordinate, does so by a different law from that for the surface of the disperse heat-transfer agent, so that their ratio is expressed by a complicated dependence. This fact introduces a new and complicating element into the solution of the problem.

As is known, the study of the kinetics of heating of the material, i.e., the determination of its temperature, which is an integral index of the process and at the same time is connected with the moisture content of the material, is very important in the drying process. Since the most pronounced variation in the temperature of the material occurs in the period of a declining drying rate, only this period is considered below.

Neglecting the gradients of temprature and moisture content over the cross section of a particle and assuming the absence of a temperature gradient over the cross section of the stream of gas suspension (i.e., developed turbulent flow of the gas suspension occurs), we

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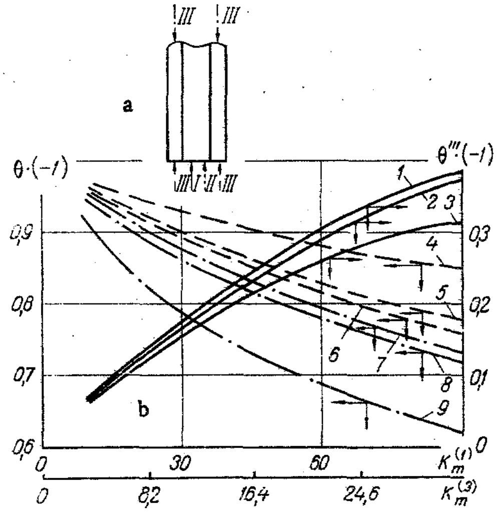

Fig. 1. Diagram of motion of heat-transfer agents and dependence of temperatures θ of heat-transfer agents on the dimensionless coefficients of heat transfer K_h: a) diagram of motion; b) solid curves θ''', dashed curves θ'', dash-dot curves θ'; curves 1, 6, 7) at $A^{(2)} = 187$, $A^{(1)} = 450$; 2, 5, 8) at $A^{(2)} = 93.5$, $A^{(1)} = 225$; 3, 4, 9) at $A^{(2)} = 23.4$, $A^{(1)} = 56$.

write the following system of equations (a system of equations for two heat-transfer agents was formulated in [4]):

$$c_1G_1dt' = -\alpha(t' - t'')\,dF_2 - k_h(t' - t''')\,dF_3, \tag{1}$$

$$(c_2 + c_w U)\,G_2dt'' - rG_2dU = \alpha(t' - t'')\,dF_2, \tag{2}$$

$$c_3G_3dt''' = \pm k_h(t' - t''')\,dF_3, \tag{3}$$

$$-\frac{dU}{dF_2} = k^*(U - U_e). \tag{4}$$

In Eqs. (1) and (3) the influence of the temperature of the heat-transfer surface (the dividing wall) is allowed for by the introduction of the heat-transfer coefficient k_h (instead of the heat-exchange coefficient). In Eq. (3) the upper sign refers to direct flow and the lower sign to counterflow. In the formulation of system (1)-(4) it was assumed that the heat exchange of the first heat-transfer agent with the second and third occurs simultaneously; direct heat exchange between the second and third heat-transfer agents does not occur.

Boundary conditions:

$$t'|_{F_2=0} = t'_{in},\quad t''|_{F_2=0} = t''_{in},\quad t'''|_{F_2=0} = t'''_{in},\quad U|_{F_2=0} = U_0; \tag{5}$$

for counterflow the condition for t''' changes:

$$t'''|_{F_2=0} = t'''_f.$$

The true surface of the disperse material inside the pipe is determined by the expression [5]

$$F_2 = nF_e^*(1 + e^{-cx}), \tag{6}$$

where $n = 1.8 + 6\gamma_m d_m$ and $c = 0.92 - 0.6\gamma_m d_m$ are empirical coefficients (d_m is in centimeters and γ_m in grams per cubic centimeter), while the calculated consumed surface of the material is determined by the expression

$$F_e^* = \mu_e \gamma_{g\,av} \sigma f_p x. \tag{7}$$

The cylindrical heat-transfer interface is $F_3 = \pi d_p x$. We introduce the dimensionless variables and parameters

$$\theta' = \frac{t' - t^i_{in}}{t^i_{in} - t''_{in}};\quad \theta'' = \frac{t'' - t^i_{in}}{t^i_{in} - t''_{in}},\quad \theta''' = \frac{t''' - t^i_{in}}{t^i_{in} - t''_{in}};\quad \xi = \frac{x}{L};$$

$$K = k^*L^2;\quad K_h^{(3)} = \frac{k_hL^2}{c_3G_3};\quad A^{(2)} = \frac{\alpha L^2}{c_2G_2};\quad A^{(1)} = \frac{\alpha L^2}{c_1G_1};\quad K_h^{(1)} = \frac{k_hL^2}{c_1G_1};\quad B_1 = \frac{b}{L};$$

$$B_2 = cL;\quad B_3 = \frac{\pi d_p}{L};\quad B_4 = \frac{r}{c_2(t^i_{in} - t''_{in})};\quad B_5 = \frac{c_w}{c_2},$$

where $b = n\mu_e\gamma_{g\,av}\sigma f_p$; $t^i_{in} = t'_{in}$ for cooling and $t^i_{in} = t'''_{in}$ for heating, it being assumed that $t''_{in} > t'_{in}$ and $t''_{in} > t'''_{in}$ for cooling and $t'''_{in} > t''_{in}$ and $t'''_{in} > t'_{in}$ for heating. Then Eqs.

(1)-(4) are rewritten as follows:

$$\begin{gathered}\frac{d\theta'}{d\xi}=-A^{(1)}[B_1(1+e^{-B_2\xi})-\xi B_1B_2e^{-B_2\xi}](\theta'-\theta'')-K_{\text{h}}^{(1)}B_3(\theta'-\theta''');\\ \frac{d\theta''}{d\xi}=[B_1(1+e^{-B_2\xi})-\xi B_1B_2e^{-B_2\xi}]\frac{[A^{(2)}(\theta'-\theta'')-KB_4(U-U_{\text{e}})]}{1+B_5U};\\ \frac{d\theta'''}{d\xi}=\pm B_3K_{\text{h}}^{(3)}(\theta'-\theta''');\\ \frac{du}{d\xi}=-K(U-U_{\text{e}})[B_1(1+e^{-B_2\xi})-\xi B_1B_2e^{-B_2\xi}].\end{gathered}\qquad(8)$$

We write the boundary conditions (direct flow) in the following form:

for heating

$$\theta'|_{\xi=0}=C\,(C\geqslant-1),\quad \theta''|_{\xi=0}=-1,\quad \theta'''|_{\xi=0}=0,\quad U|_{\xi=0}=U_0;\qquad(9)$$

for cooling

$$\theta'|_{\xi=0}=0,\quad \theta''|_{\xi=0}=-1,\quad \theta'''|_{\xi=0}=A\,(A\geqslant0),\quad U|_{\xi=0}=U_0.\qquad(10)$$

For the case of counterflow motion the final temperature of the third heat-transfer agent, i.e., $\theta'''|_{\xi=0}$, was calculated by Newton's method. Taking $z=\theta'''|_{\xi=0}$ in this case, we obtain

$$f(z)=\theta'''|_{\xi=1}.$$

Then if z_0 is the approximate value of the root of the equation $f(z)=0$, for a more precise approximation we take

$$\theta_1'''|_{\xi=0}=\theta_0'''|_{\xi=0}-\frac{f(\theta_0''')}{f'(\theta_0''')}.$$

Thus, with counterflow motion of the heat-transfer agents the boundary conditions include the temperature $\theta_n'''|_{\xi=0}$, calculated by Newton's method:

$$\theta_n'''|_{\xi=0}=\theta_{n-1}'''-\frac{f(\theta_{n-1}''')}{f'(\theta_{n-1}''')}.\qquad(11)$$

The system of differential equations (8) with conditions (9)-(11) was solved on a computer by the Runge–Kutta method. Some results of the calculations are presented in Figs. 1-4.

It is obvious that intensification of the external heat exchange, i.e., between the stream of the gas suspension and the third heat-transfer agent [an increase in the parameters $K_{\text{h}}^{(3)}$ and $K_{\text{h}}^{(1)}$] leads to an increase in the temperature of the disperse particles and of the transporting medium (the first heat-transfer agent) and to a decrease in the temperature of the high-temperature heat-transfer agent (Fig. 1b); at the same time, an increase in the intensity of heat exchange between the components, i.e., an increase in the parameters $A^{(2)}$ and $A^{(1)}$, is reflected mainly in a decrease in the temperature of the first heat-transfer agent and an increase in the temperature of the disperse particles. For example, with $A^{(2)}$ = 23.4 and $A^{(1)}$ = 56.0 for $K_{\text{h}}^{(3)}$ = 3.28 and $K_{\text{h}}^{(1)}$ = 9.0 we have t' = 73.7°C, t''' = 659.0, and t" = 37.7°C. With a fourfold increase in the intensity of external heat exchange, i.e., for $K_{\text{h}}^{(3)}$ = 13.1 and $K_{\text{h}}^{(1)}$ = 36.0, we obtain t' = 183.2°C, t" = 77.12°C, and t''' = 574.8°C; a tenfold increase in intensity ($K_{\text{h}}^{(3)}$ = 32.8 and $K_{\text{h}}^{(1)}$ = 90.0) gives t' = 280.4°C, t" = 122.7"C, and t''' = 478.3°C. We note that such intensification can be achieved by various methods, such as the use of dusty heat-transfer agents, stream swirling, replacement of a gaseous heat-transfer agent by a liquid, etc. A change in the intensity of the intercomponent heat exchange leads to the following results: for $K_{\text{h}}^{(3)}$ = 13.1 and $K_{\text{h}}^{(1)}$ = 36 with $A^{(2)}$ = 93.5 and $A^{(1)}$ = 225 (α = 150 kcal/m²·h·°C) we have t' = 128.8°C, t" = 100.9, and t''' = 560.6°C; for $A^{(2)}$ = 187.0 and $A^{(1)}$ = 450 (α = 300 kcal/m²·h·°C) we have t' = 119.96°C, t" = 105.7°C, and t''' = 557.8°C, i.e., the temperatures vary practically insignificantly, which is explained mainly by the small temperature differential between components.

From these examples it follows that intensification of the external heat exchange is the most important for the scheme of heat exchange under consideration (Fig. 1a). The data presented in Fig. 1 were obtained for the case when $U = U_e$, i.e., in the absence of moisture evaporation from the disperse material.

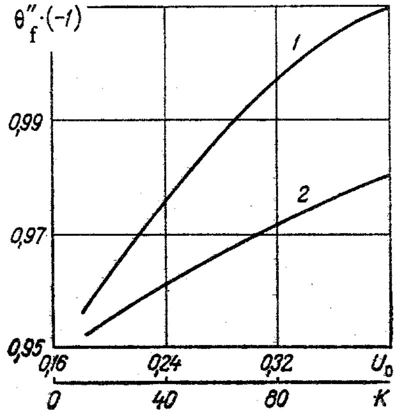

Fig. 2. Dependence of final temperature θ''_f of the material on initial moisture content U_0 (curve 1) and dimensionless drying coefficient K (2).

We note that all the calculations were made for grain with the following initial data: $L = 5$ m, $G_2 = 700$ kg/h, $G_3 = 800$ kg/h, $\mu_e = 1.75$ kg/h, $d_p = 0.08$ m, $d_m = 3.4\cdot10^{-3}$ m, $t'_{in} = 20°C$, $t''_{in} = 20°C$, $t'''_{in} = 700°C$. Grain was not chosen by chance. A number of fundamentally new and original construction solutions for grain dryers have recently appeared in which direct contact between the grain and the high-temperature smoky gases is eliminated [6, 7]. The main element of such dryers is the section of pneumatic transport of the grain in conjunction with recuperative heat supply. Therefore, the investigations made in the present work have a practical orientation. The data presented in Fig. 2 on the influence of the initial moisture content and evaporation intensity on the temperature of the material are also interesting in this connection. It follows from Fig. 2 that an increase in the initial moisture content and evaporation intensity, other conditions being equal, can lead to a decrease in the temperature of the material, which is quite understandable, since the fraction of heat going to the evaporation of the moisture increases.

Since the evaporation of moisture occurs, the influence of the mutual direction of motion of the gas suspension and the heat-transfer agent proves to be small, despite the use of a high-temeprature heat-transfer agent ($t'''_{in} = 700°C$). For example, with counterflow the temperature of the material is $\theta'' = -0.943$ (Fig. 3a, curve 6) while with direct flow it is $\theta'' = -0.951$ (Fig. 3a, curve 5); in this case $\Delta t''_f = 5.4°C$. This has a somewhat greater influence on the temperatures of the first and third heat-transfer agents: a comparison of curves 3 and 4 gives $t'''_f = 20.4°C$, while from curves 1 and 2 we get $\Delta t'_f = 57.2°C$, i.e., the process works more economically with counterflow.

Equation (4) includes an empirical coefficient k^*, the drying coefficient, which depends on many factors, particularly on the physicochemical properties of the material, its initial moisture content, and the parameters of the process. In the present work the coefficient was determined by computational—experimental means. We made variant calculations on the determination of the final temperatures θ', θ'', and θ''' and the moisture content U of the material as a function of the dimensionless drying coefficient K. Then we compared (under the same initial conditions) the experimental values of these quantities with the calculated ones; by matching the data we determined the value of K and then of k^*. The values of k^* for the case under consideration are presented in Fig. 3b.

The heat-exchange scheme under consideration also has great practical importance in the case of the cooling of disperse materials. Calculated data for the cooling of potassium chloride are presented in Fig. 4; the calculations were made for the following initial data: $L = 5$ m, $G_2 = 80\cdot10^3$ kg/h, $G_3 = 17\cdot10^3$ kg/h, $\mu_e = 3$ kg/kg, $d_p = 0.76$ m, $d_m = 3.15\cdot10^{-4}$ m, $t'_{in} = 20°C$, $t''_{in} = 110°C$, $t'''_{in} = 15°C$.

It is interesting to note that the initial moisture content has a considerable influence on the temperature of the material undergoing cooling (Fig. 4a): the final temperature of the material falls sharply with an increase in the initial moisture content (curves 1, 2, 3). Consequently, if the material must be cooled after drying, then it should not be fully dried to the final moisture content, since the process of cooling of heated moist material will occur more intensely and it can be combined with the further dehydration. The drying intensity, which can be regulated by various methods such as by the initial temperature of the

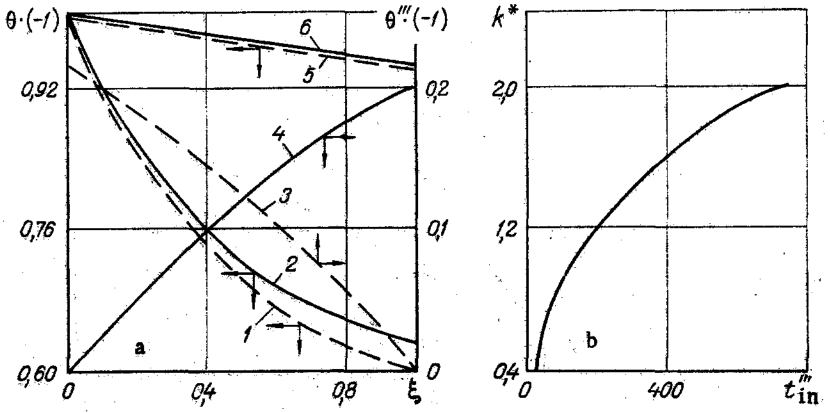

Fig. 3. Temperature distribution over the height of the pneumatic pipe and temperature dependence of the drying coefficient k^*: a: curves 1, 2) θ; 3, 4) θ'''; 5, 6) θ''; solid curves) direct flow; dashed curves) counterflow; b: dependence of k^* on t'''_{in}.

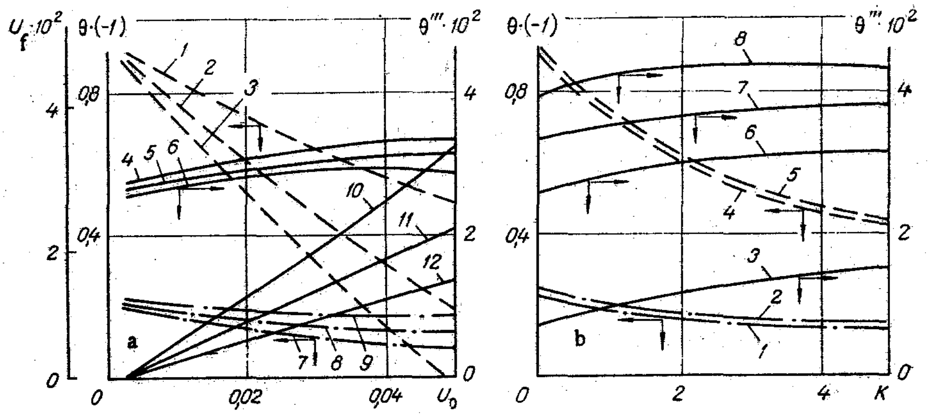

Fig. 4. Dependence of temperatures θ of heat-transfer agents and of final moisture content U_f on initial moisture content U_0 and dimensionless drying coefficient K. Solid curves) θ'''; dashed curves: θ''; dash-dot curves) θ'; a: curves 1, 6, 7) for K = 1.0; 2, 5, 8) for K = 2.0; 3, 4, 9) for K = 3.0; 10, 11, 12) for K = 1.0, 2.0, and 3.0; b: curves 1, 4, 7) for $K_h^{(3)} = 0.18$, $K_h^{(1)} = 0.32$; 2, 5, 8) for $K_h^{(3)} = 0.09$, $K_h^{(1)} = 0.16$; 6) for $K_h^{(3)} = 0.27$, $K_h^{(1)} = 0.16$; 3) for $K_h^{(3)} = 0.54$, $K_h^{(1)} = 0.32$.

transporting agent, has considerable importance in this case. Thus, when K changes from 0.1 to 5.0 the difference in the temperature of the material is 41.8°C. If one considers that the overall temperature level of the material is low in the case under consideration, then this temperature difference is considerable. Thus, by varying the initial moisture content of the material and the intensity of the drying process, one can obtain the required final temperature and moisture content of the material. The influence of the drying intensity on the final temperature of the material can be traced in more detail from Fig. 4b (curves 4 and 5). At the same time, it follows from Fig. 4a (curves 4-9) and Fig. 4b (curves 1-3, 6, 7, and 8) that the drying intensity does not have a significant effect on the temperature of the heat-transfer agents because most of the heat is expended on moisture evaporation. An analysis shows that for the given parameters the third heat-transfer agent (the cooling liquid) does not have a significant effect on the course of the cooling process and it can be used mainly for heat utilization (in the case of recovery of the cooling liquid in the technological process). For example, while intensification of the external heat exchange (a twofold increase in k_h; curves 1 and 2, 4 and 5, 3 and 6, and 7 and 8 of Fig. 4b) does lead to a change in the temperature of the material and the heat-transfer agents, this change is slight in absolute value.

Some results of the calculations are also presented in Table 1. As seen from the table, for the chosen initial data (which were determined by preliminary calculations) one can, e.g.,

TABLE 1. Values of Final Temperatures of Heat-Transfer Agents

Final temp., °C	$U_0=0{,}02$ $K_h^{(3)}=0{,}27$ $\mu_e=2$	$U_0=0{,}02$ $K_h^{(3)}=2{,}7$ $\mu_e=2$	$U_0=0{,}02$ $K_h^{(3)}=0{,}27$ $\mu_e=3$	$U_0=0{,}02$ $K_h^{(3)}=0{,}27$ $\mu_e=4$	$U_0=0{,}02$ $K_h^{(3)}=2{,}7$ $\mu_e=3$	$U_0=0{,}02$ $K_h^{(3)}=2{,}7$ $\mu_e=4$
t'	37,3	38,0	42,9	49,1	43,3	50,7
t''	86,8	86,8	87,1	87,5	87,1	87,5
t'''	8,3	25,4	8,8	9,4	28,7	32,6

considerably raise the concentration of the material without a significant change in the temperature of the material. For example, upon a twofold increase in the concentration (from μ_e = 2 to μ_e = 4 kg/kg) the temperature of the material is increased by only 0.7°C. Moreover, a tenfold decrease in the flow rate of the cooling liquid also does not significantly affect the temperature of the material. All this is explained by the fact that most of the heat is expended in moisture evaporation.

A considerable decrease in the temperature of the material can be achieved through an increase in K or in the initial moisture content of the material, which was mentioned above. Since the final moisture content of the material must not increase, most often one must increase the temperature of the transporting agent (the first heat-transfer agent) somewhat in this case, and then the use of a third heat-transfer agent proves useful, and, as a calculation shows, considerable results are achieved by using a third heat-transfer agent with a reduced initial temperature.

NOTATION

t, temperature; c, specific heat; α, coefficient of heat exchange; k_h, coefficient of heat transfer; G, flow rate; U, moisture content; U_e, equilibrium moisture content; μ_e, concentration of disperse material; d, diameter; γ, specific weight; σ, specific surface of disperse particles; f, cross-sectional area; L, pipe length. Indices: ' and 1, air; " and 2, disperse material; '" and 3, heat-transfer agent; p, pipe; m, material; g, gas; av, average; w, water.

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